



Scientific Report

AN INVESTIGATION OF THE ITE FORMULA AND ITS USE

$$CP = \left[t + \frac{V}{2a + 2Gg} \right] + \frac{W + L}{V}$$

Abstract

This working report is a study of the universally adopted ITE formula which calculates a traffic light's change interval. Its sole purpose is to provide safe passage through an intersection for a wide range of vehicle types and pedestrians with high traffic flow. However, due to misinformation and misunderstandings (both presented and found in the manuals⁵ referenced in this report) and lack of knowledge of the ITE formula's intended use with the many different State's vehicles codes, safety is compromised. Proper understanding of the basic laws of physics is needed and the Professional Engineers (PE) that are applying the ITE formula to set the timing of an intersection's traffic lights are *required by law* to understand and apply the science to provide public safety.

This report is presenting the details of how the ITE formula's terms are used to calculate the yellow and all-red phase times for vehicles traveling through an intersection with conflicting traffic and especially how to apply the formula's clearance term with the two yellow laws; the *permissive* and *restrictive* yellow laws. The report includes the needed tools to investigate and illustrate a vehicle in motion. Great effort has been taken to simplify the involved mathematics and physics. All the kinematic formulas are derived from the basic definition of the average *velocity* and *acceleration*.

The investigation shows the inherent design of the formula and that the *critical stopping distance* is the source of its design. Given by a vehicle's speed, a driver's reaction time and a safe deceleration rate; the critical stopping distance is the *only point* referenced to an intersection's entry where a driver can *either stop* safely before entering *or go* the critical distance to reach the intersection's entry point on a *legal* yellow. However, the design of the formula is not allowing the driver to slow down within the critical stopping distance and *enter* the intersection. Thus the formula is designed to *ONLY* accommodate a vehicle stopping before entering or traveling through an intersection at constant or accelerated speed. The next report will cover the yellow phase time required for turns; a time which is greater than calculated by the ITE formula due to a vehicle is slowing down *within* the *critical stopping distance* when perform a turning maneuver.

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Table of contents

Table of contents	1
1. The ITE formula.....	2
1.1 The ITE formula's three terms	2
2. The usage of the ITE formula terms.....	3
2.1 The yellow phase	3
2.2 The all-red phase	3
2.3 The permissive yellow law	3
2.4 The restrictive yellow law	3
2.4.1 Restrictive law yellow traffic light violation.....	3
2.5 Summary of the ITE formula and the two yellow traffic light laws	4
3. Perception and reaction time	5
4. Stopping and clearance time	5
5. Kinematics – The geometry of a vehicle in motion	6
5.1 Vehicle motion and the mathematics	6
5.2 Motion input variables	6
5.3 Constant deceleration or acceleration	6
5.4 Constant acceleration graphing options	7
5.5 The benefits of the velocity versus time graph	7
6. First motion example: A vehicle traveling with a constant velocity.....	8
7. Second motion example: A vehicle traveling with a constant acceleration.....	9
8. Third motion example: A vehicle traveling with a stopping motion	10
8.1 Total traveled distance calculations	10
8.2 Stopping time calculations	12
8.3 Total stopping time calculations	12
8.4 Comparison to the ITE formula	12
8.5 Why the difference?	13
9. ITE formula example using typical input values.....	13
9.1 Speed unit conversion	13
9.2 Calculation of the yellow phase time	13
9.3 Preparing to graph the example	14
9.4 Calculation of total stopping time	14
9.5 Verification of the ITE deceleration rate	14
9.6 Calculation of the total stopping distance	14
9.7 Graph of the ITE formula example	15
9.8 Graph area-distance calculations	15
9.9 Driver decisions and optional behavior	16
9.10 The ITE formula example's conclusions	16
10. Fourth motion example: A vehicle making a right hand turn.....	17
11. References.....	18
12. Appendix A - Definition of the Yellow Traffic Signal for Vehicles by State	19
13. Appendix B - Emergency Stopping Distances and Time Calculations (Rev. 10)	24
14. Appendix C - Deceleration Rates and Stopping Distances Comparison (Rev. 5)	25

This report is dedicated to Marianne Järnlström and David Hodge.

1. The ITE formula

The Institute of Transportation Engineers' ITE formula was developed by Denos Gazis from GM Research Labs, Robert Herman and Alexei Maradudin and presented in 1959 in the paper "*The Problem of the Amber Signal Light in Traffic Flow*"¹. Today the formula is used worldwide to calculate traffic light phase times such as the yellow change and all-red clearance intervals. Here is one example of this formula^{1 2 3 4 5}:

$$CP = \left[t + \frac{V}{2a + 2Gg} \right] + \frac{W + L}{V} \quad (1.1)$$

Where:

CP = Change Period, total combined driver perception and reaction, vehicle stopping and clearance times, result expressed in seconds, (s).

t = Perception and reaction time of the driver, typically 1.0 seconds for an expected event, (s).

V = Speed of the approaching vehicle, expressed in feet per second, (ft/s).

a = Comfortable deceleration rate of the vehicle, typically 10 feet per second squared, (ft/s²).

W = Width of the intersection at widest conflict point, expressed in feet, (ft).

L = Length of vehicle, typically 20 feet, (ft).

G = Acceleration due to gravity, 32.2 feet per second squared, (ft/s²).

g = Grade of the intersection approach, in percent (%) divided by 100, downhill is negative grade and uphill is positive grade.

1.1 The ITE formula's three terms

By studying the ITE formula (1.1) and the individual input variables, we can determine that it consists of three terms and all terms appear to specify or calculate time in seconds as follows:

1. Perception and reaction time of the driver = t (1.2)

2. Deceleration time of the vehicle = $\frac{V}{2a + 2Gg}$ (1.3)

3. Intersection and vehicle clearance time = $\frac{W + L}{V}$ (1.4)

Describing the ITE formula (1.1) and its terms, the equation is simplified as follows:

$$CP = \left[t + \frac{V}{2a + 2Gg} \right] + \frac{W + L}{V}$$

or

$$\text{Change Period} = \left[\begin{array}{c} \text{Perception} \\ \text{Reaction} \\ \text{Time} \end{array} + \begin{array}{c} \text{Deceleration} \\ \text{Time} \end{array} \right] + \begin{array}{c} \text{Intersection \& Vehicle} \\ \text{Clearance Time} \end{array}$$

or

$$\text{Change Period} = \left[\begin{array}{c} \text{Total Stopping Time} \end{array} \right] + \begin{array}{c} \text{Clearance Time} \end{array}$$

2. The usage of the ITE formula terms

This section explains how the three ITE formula terms are currently used ² to implement the timing of traffic lights with different State's vehicle codes ⁵ presented in APPENDIX A.

2.1 The yellow phase

The driver perception and reaction time (1.2) and the deceleration time (1.3) terms are typically combined to calculate a traffic light's yellow phase time which is also the total stopping time of the ITE formula. This total stopping time is also directly linked to the "*one safe stopping distance*" or Gazis' "*critical stopping distance*" which will be further investigated later in this report.

2.2 The all-red phase

The remaining term, the intersection and vehicle clearance time (1.4), is commonly used to calculate traffic light all-red phase times. The all-red phase is a clearance time when all traffic lights are red and no vehicles are allowed to enter the intersection from any of its approaches. The all-red phase time allow vehicles that are still in the intersection to exit before conflicting traffic, including pedestrians that are given a green light to enter. The clearance term adds an important safety time to avoid traffic accidents.

2.3 The permissive yellow law

The *permissive yellow light law* is when a State's vehicle code only *warns* that a change of the traffic light from yellow to red is imminent and is thereby *permitting* a driver to enter the intersection during the *full* yellow phase. For this law, the all-red phase is mandatory since a vehicle can legally enter the intersection at the very end of the yellow phase and thus needs time to drive through and exit the intersection during the protection of the all-red phase. A violation occurs if the driver enters the intersection on a red traffic light signal.

2.4 The restrictive yellow law

There is also a *restrictive yellow light law* where a driver facing a yellow light "*shall stop*" and not enter the intersection unless the driver "*cannot stop in safety*". A driver cannot stop in safety if the driver is closer to the intersection than "*one safe stopping distance*" or the "*critical stopping distance*".

Some State's restrictive yellow light vehicle codes also add instructions for a driver's optional behavior when facing the yellow light such as "*if a driver cannot stop in safety, the driver may cautiously drive through the intersection*". Here, the "*drive through the intersection*" is the ITE formula's clearance term (1.4) which for a *restrictive yellow light law* is added to the traffic light's yellow phase time. In addition, the word "*cautiously*" is instructing the driver not to accelerate to reach and clear the intersection's exit. Any unsafe acceleration would also violate the speed limit if the driver approached the intersection at the speed limit. For the *restrictive yellow light law* the all-red phase is *optional* since the clearance time is already included in the yellow phase time.

2.4.1 Restrictive law yellow traffic light violation

A jurisdiction having the restrictive yellow light vehicle code can cite a driver running a yellow light because the yellow phase time *includes the clearance term* (1.4) and is therefore longer than just the ITE formula's total stopping time. The words "*shall stop*" used by the restrictive yellow law is specifically added to prohibit or restrict the driver to use the *added* clearance time to enter the

intersection. Thus a citation can be issued if the driver enters the intersection during the *added* yellow clearance time. (See also the marked "Violation Area" in figure 1).

2.5 Summary of the ITE formula and the two yellow traffic light laws

To summarize the ITE formula's terms and their usage with the *permissive* and the *restrictive yellow light laws* we have:

The Permissive Yellow Law

Where the driver is *permitted* to enter the intersection during the *full* yellow phase.

$$\text{Yellow Phase Time} = t + \frac{V}{2a + 2Gg} \quad \text{All - Red Phase Time} = \frac{W + L}{V}$$

"Total Stopping" "Clearance"

The Restrictive Yellow Law

Where the "driver shall stop facing the light" due to the clearance time is added to the yellow phase.

$$\text{Yellow Phase Time} = t + \frac{V}{2a + 2Gg} + \frac{W + L}{V} \quad \text{All - Red Phase Time} = \text{Optional}$$

"Total Stopping + Clearance"

The below figure 1 illustrates the ITE formula terms and the two yellow traffic light laws in a scaled intersection showing relative traffic light phase times for a constant velocity vehicle. The timing graphs of the traffic lights also show how the all-red phase relates to the conflicting traffic signal and when a traffic light violation occurs with the two different laws:

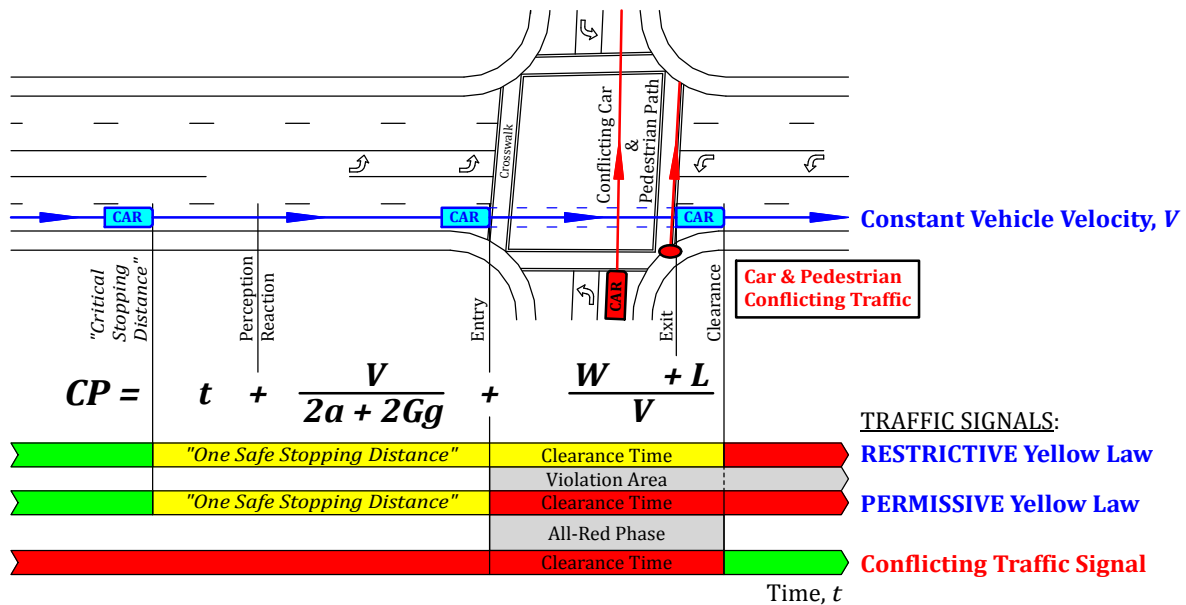


Fig. 1 - The ITE Formula Relative a Traffic Light Intersection and the Two Yellow Light Laws

3. Perception and reaction time

Driver perception and reaction time ⁶⁷ is a time where no changes are taking place to a vehicle's motion. This is due to it takes a driver some time to perceive and react to, for example a traffic signal changing from green to yellow and to make a decision whether he or she should make any changes such as stop or go. The time it takes can be broken down into three categories depending on what type of event the driver is reacting to or is making. Three low complexity type of events and some typical perception and reaction times used by ITE with examples are as follows:

1. Unexpected external event: 2.5 seconds - A deer entering the roadway.
2. Expected external event: 1.0 seconds - A changing traffic light or traffic control device.
3. Planned internal event *: 0.0 seconds - A driver is making a lane change or a turn.

* Event introduced by author.

Note: A traffic light is considered an expected event but an incorrectly timed traffic light intersection can cause unexpected events such as pedestrian or vehicle interferences. In addition, different vehicle braking systems such as tractor-trailer, school and public bus air brakes will add an extra reaction time delay of 0.5 seconds or more ⁸.

4. Stopping and clearance time

By studying the stopping and clearance terms of the ITE formula, we see the following input variables; vehicle length, vehicle speed and vehicle deceleration, plus intersection grade and intersection clearance width. Distance, velocity and acceleration can be presented in graph form to help us visualize and investigate the true physical nature of the ITE formula. The next step is to introduce visual tools such as vehicle motion graphs.

Note: Since this is working report, next version will included more detailed studies of the individual input variables of ITE formula in this section. Appendix B and C are included at this time which present some of this information:

Appendix B is presenting the effects of different stopping distances based on maximum roadway friction (emergency stopping) ⁶⁷ and air brake delays needed by trucks, public and school busses ⁸.

Appendix C is presenting a collection of maximum decelerations rates for different vehicle types and also their "cargo" which includes bus passengers and the related stopping distances.

5. Kinematics – The geometry of a vehicle in motion

The goal is to investigate the ITE formula and how it relates to a vehicle's motion in time and space by using basic mathematics and also present its motion using visual graphing tools.

5.1 Vehicle motion and the mathematics

A vehicle has three tight-coupled variables of motion; distance (d), velocity (V) and acceleration (a). The below flow diagram in figure 2 illustrates these states of motion and how they are linked through mathematical calculus functions which are called differentiation and integration. Differentiation is looking at a plotted curve's slope and integration is looking at the area under a plotted curve. However, this document is going to present a simplified method to use "calculus" to analyze the ITE formula by avoiding advanced mathematics on curves.

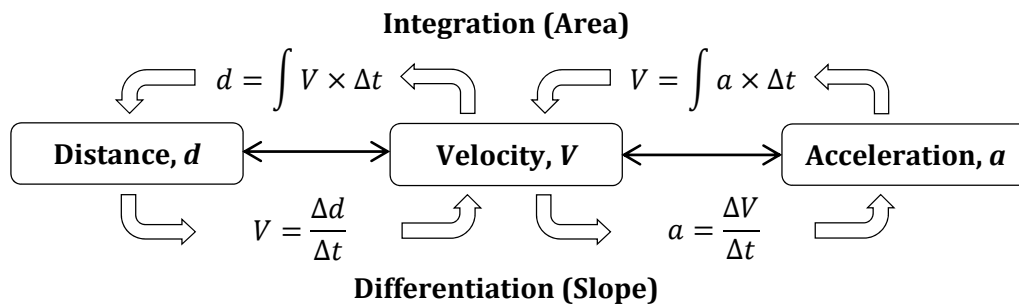


Fig. 2 - Flow Diagram of Motion over Time and their Mathematical Relationships

Figure 2 presents that the three variables of motion are closely connected through mathematics. We can mathematically convert, for example, acceleration to velocity by integrating acceleration over elapsed time (Δt), (the symbol " Δ " represents "change"). We can also convert distance over time to velocity by using differentiation. Using words, we can also describe differentiation and how it relates to a driver of a vehicle: The vehicle's velocity is the first derivative of the distance. Stepping on the accelerator or the brake, we experience a second derivative - acceleration or deceleration.

5.2 Motion input variables

We are familiar with both distance and velocity since most vehicles are equipped with both an odometer for distance and a speedometer for speed. Typically we have no standard "meter" installed in our cars to measure acceleration, even though "g-meters" are popular as an accessory for performance car enthusiasts.

In the United States, vehicle odometers measure distance in miles and the speedometers measure velocity in miles per hour (mph). As a driver, we continuously monitor the instantaneous vehicle speed (V), if not, we might get a speeding citation.

5.3 Constant deceleration or acceleration

The ITE formula is using vehicle velocity (V) as one important input variable. The formula is also including a *constant* deceleration rate (a) defined with a typical value of 10 ft/s². This *constant* deceleration rate (a) is telling us how fast a vehicle is slowing or is able to slow down or stop. One important factor to understand is that the ITE formula's average deceleration rate is a *constant rate or value over time* and can easily be plotted in a graph.

5.4 Constant acceleration graphing options

Let us look at the graphing options based on an average *constant acceleration* (a) and see how the closely related velocity (V) and distance (d) are visually presented versus time.

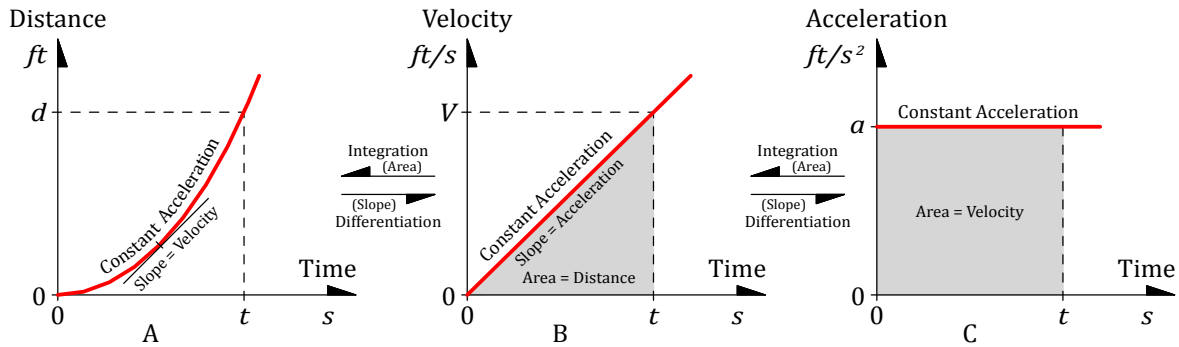


Fig. 3 - Graphing Options for Motion with Constant Acceleration

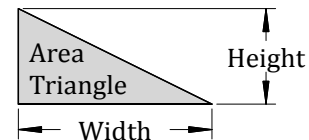
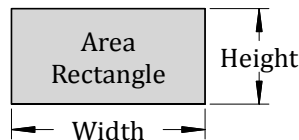
Figure 3 shows how the average constant acceleration (a) is plotted using the three variables of motion versus time. Studying the above graphs in figure 3 A, B and C we see:

- A. Distance (d) versus time (t) graph shows constant acceleration (a) plotted as a curve.
- B. Velocity (V) versus time (t) graph presents the constant acceleration (a) as a straight line raising over time.
- C. Acceleration (a) versus time (t) graph represents the constant acceleration (a) as a straight horizontal line.

We can also see in figure 3 that some areas under the plotted lines are shaped as triangles or rectangles. We also know that we can mathematically transform, for example, acceleration to velocity or velocity to distance using the calculus function called integration. Integration is the same as computing the area under a plotted curve. By carefully choosing a graphing method that will avoid curves and only uses straight lines we can simplify the mathematics for the “integration” or area calculations to basic geometry area calculations of rectangles and triangles:

$$\text{Area of a rectangle} = \text{Height} \times \text{Width}$$

$$\text{Area of a triangle} = \frac{\text{Height} \times \text{Width}}{2}$$



For example in figure 3C, velocity (V) is the integration of acceleration (a). Thus, integration is the area under the plotted line in the acceleration versus time graph which is equal to the “height” constant acceleration (a) times the “width” elapsed time (Δt).

5.5 The benefits of the velocity versus time graph

From the three graphing options we can see that by choosing a velocity (V) versus time (t) graph we get these key benefits:

- 1. The constant acceleration (a) defined by the ITE average deceleration rate, is velocity (V) plotted as a straight line in a velocity versus time graph.

2. Single integration which is the area under the plotted line of the velocity versus time graph will calculate traveled distance (Δd) during the elapsed time (Δt).
3. The integration of the velocity versus time plot becomes simple since the graph only have straight lines and we can use basic mathematics such as area and geometry calculations of rectangular and triangular shapes. Thus avoiding using advanced calculus functions or mathematics on curves.
4. Velocity or vehicle speed is the instantaneous measurement we as drivers are most familiar with.

Before we graph the ITE formula itself we can start to look at simple vehicle motion profiles using this graphing method and see with examples how vehicle velocity or speed versus time relate to distance and acceleration and their corresponding mathematical formulas.

6. First motion example: A vehicle traveling with a constant velocity

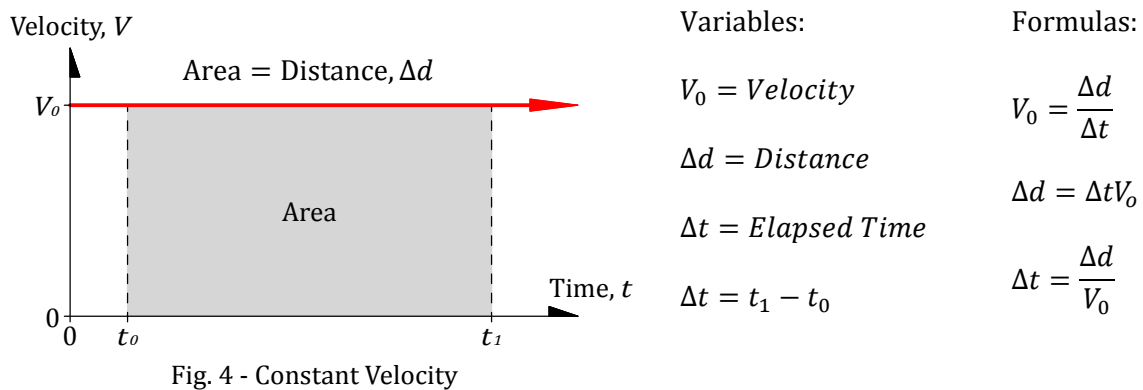


Figure 4 shows a velocity versus time graph of a vehicle traveling at a constant speed V_0 from time t_0 to time t_1 . The constant speed is represented as a straight horizontal line over time. Speed or velocity is defined as distance traveled over elapsed time as we also see in the units we use for speed such as miles per hour (mph).

Based on the definition of average velocity we have:

$$\text{Average velocity, } V_0 = \frac{\text{Distance, } \Delta d}{\text{Elapsed time, } \Delta t} \quad (6.1)$$

Rearranging above equation (6.1) we also get:

$$\text{Distance, } \Delta d = \Delta t V_0 \quad (6.2)$$

And:

$$\text{Elapsed time, } \Delta t = \frac{\Delta d}{V_0} \quad (6.3)$$

We can also see that the area under the graph is the height (velocity, V_0) multiplied with the width (elapsed time, $\Delta t = t_1 - t_0$). This area under the graph is the same as the traveled distance (Δd) of the vehicle as shown in equation (6.2). This visual understanding that the area under the plotted line in a velocity versus time graph equals distance will be very useful when we start to look at more complex motion profiles with changing vehicle speeds over time. This change of velocity over time is also referred to as acceleration or deceleration.

7. Second motion example: A vehicle traveling with a constant acceleration

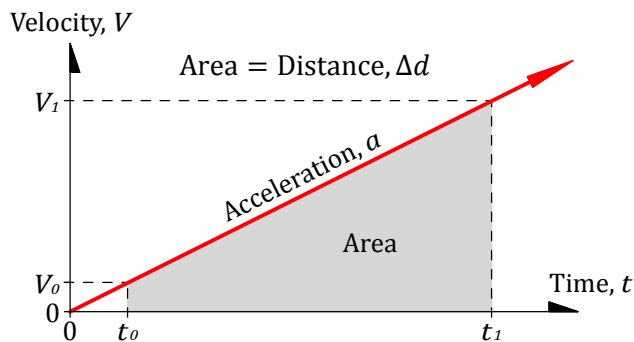


Fig. 5 - Constant Acceleration

Variables:

$a = \text{Acceleration}$

$V_0 = \text{Velocity at } t_0$

$V_1 = \text{Velocity at } t_1$

$\Delta d = \text{Distance}$

$\Delta t = \text{Elapsed Time}$

$\Delta t = t_1 - t_0$

Formulas:

$$a = \frac{V_1 - V_0}{\Delta t}$$

$$\Delta t = \frac{V_1 - V_0}{a}$$

$$V_1 = V_0 + a\Delta t$$

$$\Delta d = \Delta t \left(\frac{V_1 + V_0}{2} \right)$$

$$\Delta d = \frac{V_1^2 - V_0^2}{2a}$$

Figure 5 shows a velocity versus time graph of a vehicle accelerating at a constant rate (a) from a standstill. At time t_0 the vehicle has reached a speed V_0 and at time t_1 the vehicle has reached speed V_1 . The average acceleration (a) of the vehicle is defined as change in velocity ($V_1 - V_0$) over elapsed time ($\Delta t = t_1 - t_0$).

The definition of average acceleration is:

$$\text{Average acceleration, } a = \frac{\text{Change in Velocity, } V_1 - V_0}{\text{Elapsed time, } \Delta t} \quad (7.1)$$

When the vehicle speed is increasing over time, as presented in figure 5, the term $V_1 - V_0$ in formula (7.1) becomes positive and we have positive acceleration. If the vehicle speed is decreasing over time the term $V_1 - V_0$ becomes negative and we get negative acceleration. Negative acceleration is also called deceleration and occurs when the vehicle is slowing down or stopping.

Rearranging above equation (7.1) we also get:

$$\text{Elapsed time, } \Delta t = \frac{V_1 - V_0}{a} \quad (7.2)$$

And:

$$\text{Velocity, } V_1 = V_0 + a\Delta t \quad (7.3)$$

Note: As figure 5 shows, V_1 represents end velocity and V_0 initial velocity.

In figure 5, the area under the graph, which is also the distance the vehicle is traveling during the elapsed time ($\Delta t = t_1 - t_0$), is not as easy to calculate as with the previous constant velocity vehicle example. To solve the problem we will look at the two speed values at time t_0 and t_1 and calculate the average velocity. The average velocity is simple the sum of $V_1 + V_0$ divided by 2. The area under the curve is then the average “height” or velocity times the elapsed time “width”.

The formula for the traveled distance (Δd) during the elapsed time (Δt) is then:

$$\text{Distance, } \Delta d = \Delta t \left(\frac{V_1 + V_0}{2} \right) \tag{7.4}$$

If we do not know the time (Δt) we can combine the above distance formula (7.4) with the formula for elapsed time (7.2) as shown here:

Take (7.2) $\Delta t = \frac{V_1 - V_0}{a}$ and combine with (7.4) $\Delta d = \Delta t \left(\frac{V_1 + V_0}{2} \right)$ which gives:

$$\text{Distance, } \Delta d = \frac{V_1^2 - V_0^2}{2a} \tag{7.5}$$

Hint: Use the conjugate rule $(a + b)(a - b) = a^2 - b^2$ to combine the above formulas.

8. Third motion example: A vehicle traveling with a stopping motion

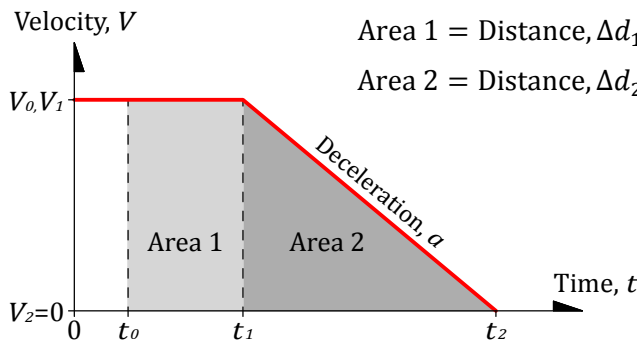


Fig. 6 - Constant Velocity and Deceleration

Area 1 = Distance, Δd_1
 Area 2 = Distance, Δd_2

Area 1 formulas:

$$\text{Time, } \Delta t_1 = (t_1 - t_0) = \frac{\Delta d_1}{V_0}$$

$$\text{Distance, } \Delta d_1 = \Delta t_1 V_0$$

Area 2 formulas:

$$\text{Time, } \Delta t_2 = (t_2 - t_1) = \frac{V_1}{a}$$

$$\text{Distance, } \Delta d_2 = \Delta t_2 \left(\frac{V_1}{2} \right)$$

$$\text{Distance, } \Delta d_2 = \frac{V_1^2}{2a}$$

Figure 6 shows a vehicle traveling with an initial constant velocity (V_0) up until time t_1 . The velocity (V_1) at time t_1 is still V_0 so we have $V_0 = V_1$. From t_1 the vehicle is decelerating at a constant rate (a) to a complete stop at time t_2 . If we also introduce an initial time t_0 which is an added time before the vehicle is decelerating we see that this is a vehicle motion profile that is taking the shape of the first two terms of the ITE formula (1.1) – driver perception and reaction time ($t_1 - t_0$) plus vehicle stopping time ($t_2 - t_1$).

8.1 Total traveled distance calculations

The ITE formula’s first term is the driver perception and reaction time. In figure 6 we can set the elapsed time (Δt_1) between time t_0 to t_1 to be the driver perception and reaction time value. Area 1

is then the distance the vehicle would travel during this perception and reaction time. We can use the first motion example and use its information with formula (6.2) since between time t_0 to t_1 both motion example vehicles are traveling at a constant velocity. The area 1 and distance (Δd_1) traveled during the perception and reaction time ($\Delta t_1 = t_1 - t_0$) is then:

$$\Delta d_1 = (t_1 - t_0)V_0 \quad \text{or} \quad \Delta d_1 = \Delta t_1 V_0 \quad (8.1)$$

From time t_1 to t_2 , figure 6 is showing a vehicle's motion to slow down to a complete stop. At time t_1 the vehicle's speed is V_1 and it starts to decelerate with an average negative acceleration (a) until it has come to a complete stop at time t_2 . Area 2 is the distance (Δd_2) the vehicle is traveling during the stopping or deceleration to come to a complete stop.

To calculate area 2 which is the distance (Δd_2) in figure 6 we can use the same methods and formulas as in the second motion example where the vehicle change velocity over time. In this example, the vehicle is decelerating to a complete stop so at time t_2 the velocity is zero ($V_2=0$). Therefore, the two distance formulas (7.4) and (7.5) then become:

$$\text{Area 2} = \Delta d_2 = (t_2 - t_1) \left(\frac{V_2 + V_1}{2} \right) \quad \text{or} \quad \text{Area 2} = \Delta d_2 = \frac{V_2^2 - V_1^2}{2a}$$

Set $V_2=0$ (since vehicle stopped completely) and we get:

$$\Delta d_2 = (t_2 - t_1) \frac{V_1}{2} \quad \text{or} \quad \Delta d_2 = \frac{-V_1^2}{2a}$$

Since (a) is deceleration or negative acceleration, we change the sign of (a) to get:

$$\Delta d_2 = (t_2 - t_1) \frac{V_1}{2} \quad \text{or} \quad \Delta d_2 = \frac{V_1^2}{2a}$$

The total traveled distance (Δd) in figure 6 from time t_0 to time t_2 is then:

$$\begin{aligned} \Delta d &= \Delta d_1 + \Delta d_2 & \text{and set} & \quad V = V_0 = V_1 \\ \Delta d &= (t_1 - t_0)V + (t_2 - t_1) \frac{V}{2} & \text{or} & \quad \Delta d = (t_1 - t_0)V + \frac{V^2}{2a} \end{aligned}$$

If we compare the ITE formula with the two above distance equations we see that the equation including the deceleration term (a) is the best choice due to this variable is part of the ITE formula as one of the specified input values. We can also simplify the formula by using the variable (t) for the driver perception and reaction time instead of the elapsed time ($\Delta t_1 = t_1 - t_0$). The total stopping distance formula for this motion example then becomes:

$$\text{Total Stopping Distance, } \Delta d = tV + \frac{V^2}{2a} \quad (8.2)$$

The above distance formula (8.2) is also the ITE "one safe stopping distance" or the "critical stopping distance" if the vehicle is stopping on a level approach grade ($g=0$), traveling at a constant approach speed (V), setting the driver perception and reaction time to (t) and the road conditions and vehicle brakes will allow a deceleration rate (a).

8.2 Stopping time calculations

Let us now take a look at the time it takes for the vehicle to decelerate from time t_1 to t_2 in figure 6. In the second motion example we studied acceleration and we used the definition of the average acceleration to derive the elapsed time formula (7.2) again seen here:

$$\Delta t = \frac{V_1 - V_0}{a}$$

In the current example we are using different references to the initial velocity and the end velocity. We can rewrite formula (7.2) to match this example's area 2 as follows:

$$\Delta t_2 = t_2 - t_1 = \frac{V_2 - V_1}{a}$$

We already set $V_2=0$ in this example since the vehicle has completely stopped at time t_2 . We also know that the average acceleration (a) is negative since the vehicle is decelerating. Setting $V_2=0$ and changing the sign of variable (a) to represent deceleration instead of acceleration we get:

$$\Delta t_2 = t_2 - t_1 = \frac{V_1}{a} \quad (8.3)$$

The above formula (8.3) calculates the stopping time of the vehicle in figure 6 which is decelerating at rate of (a) to a complete stop from an initial velocity of (V_1).

8.3 Total stopping time calculations

By adding the driver perception-reaction time (t) (Area 1, $\Delta t_1 = t_1 - t_0$ in figure 6) to formula (8.3) we get the total stopping time Δt from time t_0 to time t_2 . Thus, this formula would calculate the time it takes for a vehicle to travel "one safe stopping distance" or the "critical stopping distance" as per equation (8.2). Adding the perception-reaction time (t) to formula (8.3) we get:

$$\text{Total Stopping Time, } \Delta t = (t_2 - t_0) = t + \frac{V}{a} \quad (8.4)$$

8.4 Comparison to the ITE formula

Let us now compare equation (8.3) for the vehicle's stopping or deceleration time in figure 6 with the ITE formula's second term (1.3) which is calculating the vehicle's deceleration time used for yellow traffic light change intervals.

Derived deceleration time formula (8.3):

$$\Delta t_2 = t_2 - t_1 = \frac{V_1}{a}$$

Set $V_1=V$ (vehicle approach speed) and we get

$$\frac{V}{a}$$

ITE formula deceleration time term (1.3):

$$\frac{V}{2a + 2Gg}$$

If grade is level, set $g=0$ and we get

$$\frac{V}{2a}$$

The above comparison show that the derived stopping time formula (8.3) is NOT matching the ITE formula's deceleration term (1.3) and the time it takes to decelerate to zero from an initial speed (V) for a given deceleration (a).

8.5 Why the difference?

The ITE formula's deceleration term (1.3) show an extra "2" in its denominator compared to formula (8.3) which is effectively doubling the deceleration rate (a) or dividing the vehicle's approach speed (V) by 2. Fact is, the ITE expression will reduce the calculated stopping time by a factor of two. We need to investigate why this "2" is added and also what effects it has to the timing of a traffic light's change interval related to a vehicle's motion.

It is now time to use all the derived formulas and the visual graphing tools by calculating an actual example using the typical input values recommended by the US Federal Highway Administration and the international Institute of Transportation Engineers.

9. ITE formula example using typical input values

For this example we will calculate the yellow traffic light's stopping time in a *permissive* State (no clearance time added to the yellow phase) for a 30 mph approach speed at a level intersection. The ITE formula and the input values are as follows:

$$\text{Yellow Phase Time} = t + \frac{V}{2a + 2Gg} \quad (9.1)$$

Where:

t = Perception and reaction time of the driver, typically 1.0 seconds for an expected event, (s).

V = Speed of the approaching vehicle, expressed in feet per second, (ft/s).

a = Comfortable deceleration rate of the vehicle, typically 10 feet per second squared, (ft/s²).

G = Acceleration due to gravity, 32.2 feet per second squared, (ft/s²).

g = Grade of the intersection approach, in percent (%) divided by 100, downhill is negative grade and uphill is positive grade.

9.1 Speed unit conversion

First we need to convert the 30 mph vehicle approach speed to ft/s so we work with the correct units. To do this conversion we look at the unit mph which is miles per hour. We know that one mile is 5280 feet. We also know that one hour is sixty minutes and one minute is sixty seconds so we can setup the mph to ft/s conversion like this:

$$1 \text{ mph} = 1 \frac{\text{miles}}{\text{hour}} = \frac{5280 \text{ feet}}{60 \text{ minutes}} = \frac{5280 \text{ feet}}{60 \times 60 \text{ seconds}} = \frac{5280 \text{ ft}}{3600 \text{ s}} = 1.466667 \text{ ft/s}$$

The example has an approach speed of 30 mph and if we apply the mph to ft/s conversion constant we get:

$$V = 1.466667 \frac{\text{ft/s}}{\text{mph}} \times 30 \text{ mph} = 44 \text{ ft/s}$$

9.2 Calculation of the yellow phase time

Next, we can add the input values to the ITE formula for the example calculation:

$$\text{Yellow Phase Time} = t + \frac{V}{2a + 2Gg} = 1.0 \text{ s} + \frac{44 \text{ ft/s}}{2 \times 10 \text{ ft/s}^2} = 1.0 \text{ s} + 2.2 \text{ s} = 3.2 \text{ s}$$

Note: The term "2Gg" becomes zero since this example has a level approach grade ($g=0$).

Let us now plot the example in a velocity versus time graph using the typical ITE formula input values and also visually present the above calculated traffic light's yellow phase time of 3.2 seconds referenced to the vehicle's motion profile.

9.3 Preparing to graph the example

To plot the example we should first investigate the data to set the velocity and time scales appropriately. Here is an initial list of the key events or data points to plot:

- Vehicle approach speed, $V=44 \text{ ft/s}$ (30 mph)
- Driver perception-reaction time, $t=1.0 \text{ s}$
- Vehicle deceleration rate, $a=10 \text{ ft/s}^2$
- Yellow phase time= 3.2 s

9.4 Calculation of total stopping time

The previous list present a maximum velocity of 44 ft/s and a maximum ITE yellow phase time of 3.2 seconds. Let us investigate the vehicle's stopping time, Δt based on the approach speed (V) and deceleration (a) using information and the derived formula (8.3) from the third motion example. Adding the example values we get:

$$\text{Vehicle Stopping Time, } \Delta t = \frac{V}{a} = \frac{44 \text{ ft/s}}{10 \text{ ft/s}^2} = 4.4 \text{ s}$$

If we can also calculate the total stopping time using formula (8.4) which includes the 1.0 seconds driver perception-reaction time (t) and we have:

$$\text{Total Vehicle Stopping Time, } \Delta t = t + \frac{V}{a} = 1.0 \text{ s} + \frac{44 \text{ ft/s}}{10 \text{ ft/s}^2} = 5.4 \text{ s}$$

Based on this we see that the horizontal time scale should be a minimum of 5.4 seconds.

9.5 Verification of the ITE deceleration rate

We can also check that the ITE deceleration rate, $a=10 \text{ ft/s}^2$ is correct by using the formula for the definition of acceleration (7.1). Set vehicle stopping time, $\Delta t=4.4 \text{ s}$, vehicle approach speed, $V_0=44 \text{ ft/s}$ and $V_1=0$ since the vehicle comes to a complete stop in formula (7.1):

$$\text{Average acceleration, } a = \frac{\text{Change in Velocity, } V_1 - V_0}{\text{Elapsed time, } \Delta t}$$

Add values plus change acceleration to deceleration (change sign of V_0 and set $V_1=0$) and we get:

$$a = \frac{V_1 - V_0}{\Delta t} = \frac{0 - 44 \text{ ft/s}}{4.4 \text{ s}} = -10 \text{ ft/s}^2$$

The above result shows that the stopping time, $\Delta t=4.4 \text{ s}$ and the ITE deceleration rate, $a=10 \text{ ft/s}^2$ are verified correctly at 44 ft/s (30 mph) vehicle speed.

9.6 Calculation of the total stopping distance

Let us also calculate the example vehicle's total stopping distance which is also including the distance the vehicle is traveling during the driver perception-reaction time. Here we can use the "one safe stopping distance" or the "critical stopping distance" formula (8.2) which was derived in the third motion example.

The formula and adding the example values we get:

$$\Delta d = tV + \frac{V^2}{2a} = 1.0 \text{ s} \times 44 \text{ ft/s} + \frac{(44 \text{ ft/s})^2}{2 \times 10 \text{ ft/s}^2} = 44 \text{ ft} + 96.8 \text{ ft} = 140.8 \text{ ft}$$

The calculated "critical stopping distance" of 140.8 feet is the distance it takes for a vehicle to stop if it is traveling at 30 mph and the driver takes one second to react and respond to a change of a traffic control device based on the comfortable or nonemergency ITE deceleration rate of 10 ft/s².

We now have all information needed to plot the example with values.

9.7 Graph of the ITE formula example

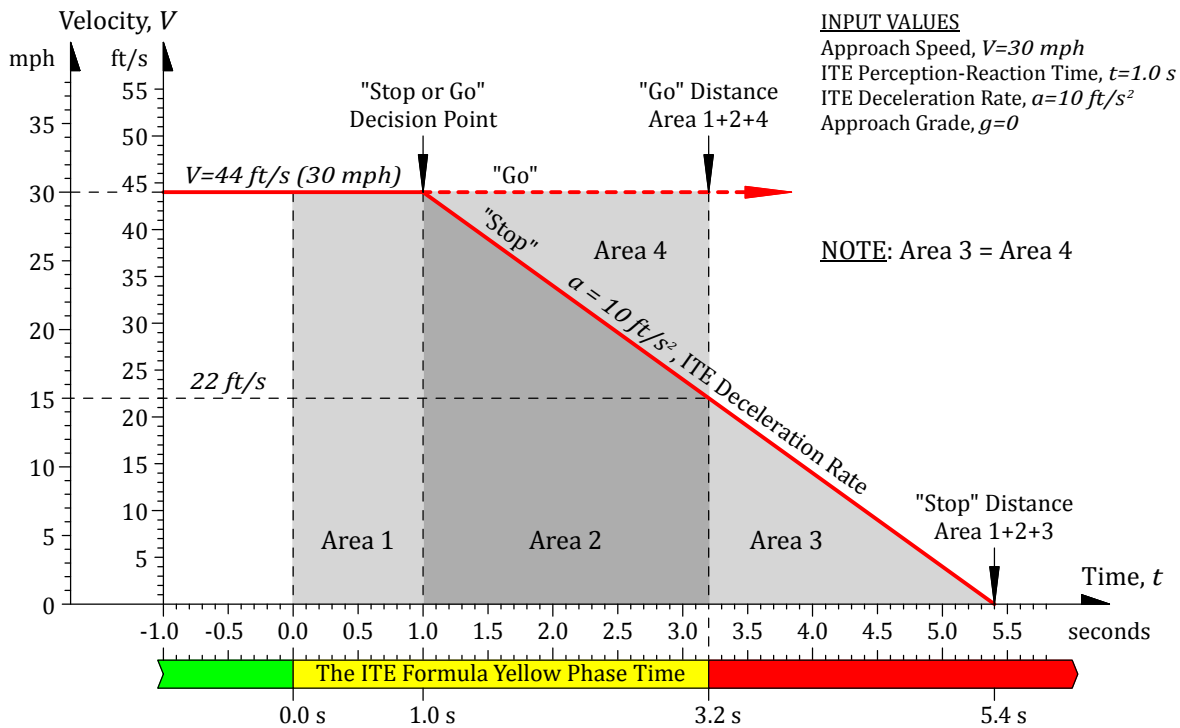


Fig. 7 - ITE Formula Example - 30 MPH Vehicle Motion and The Yellow Phase

9.8 Graph area-distance calculations

Figure 7	Average Velocity, V (Height)	Elapsed Time, Δt (Width)	Distance, Δd (Area)
Area 1:	44 ft/s	$\times 1.0 \text{ s}$	$= \Delta d_1=44.0 \text{ ft}$
Area 2:	$\frac{44 \text{ ft/s} + 22 \text{ ft/s}}{2} = 33 \text{ ft/s}$	$\times 2.2 \text{ s}$	$= \Delta d_2=72.6 \text{ ft}$
Area 3:	$\frac{22 \text{ ft/s} + 0 \text{ ft/s}}{2} = 11 \text{ ft/s}$	$\times 2.2 \text{ s}$	$= \Delta d_3=24.2 \text{ ft}$
Area 4:	$\frac{0 \text{ ft/s} + 22 \text{ ft/s}}{2} = 11 \text{ ft/s}$	$\times 2.2 \text{ s}$	$= \Delta d_4=24.2 \text{ ft}$

Summary of calculated distance results from figure 7

Driver perception-reaction distance:	(Area 1)	$\Delta d_1 = 44.0 \text{ ft}$
Vehicle "Stop" distance:	(Area 2+3)	$\Delta d_2 + \Delta d_3 = 96.8 \text{ ft}$
Total "Stop" distance:	(Area 1+2+3)	$\Delta d_1 + \Delta d_2 + \Delta d_3 = 140.8 \text{ ft}$
Total "Go" distance:	(Area 1+2+4)	$\Delta d_1 + \Delta d_2 + \Delta d_4 = 140.8 \text{ ft}$

9.9 Driver decisions and optional behavior

Figure 7 shows a vehicle traveling at a speed of 30 mph or 44 ft/s. At time 0.0 seconds the driver sees a change of a traffic control device - a traffic signal is changing from green to yellow.

The driver takes 1.0 seconds to perceive and react to the traffic light's phase change. During the driver perception and reaction time of 1.0 seconds the vehicle is traveling 44 ft (Area 1). After this time the driver shall have decided to either "stop" or "go" as follows:

- **"Stop" decision**

At 1.0 seconds the driver decides to stop and the vehicle is decelerating at the typical rate of 10 ft/s². It takes 4.4 seconds to decelerate to a complete stop and during the deceleration the vehicle is traveling 96.8 ft (Area 2 and 3 in figure 7). The total time and distance traveled (including the distance the vehicle traveled during the 1 second driver perception-reaction time) is 5.4 seconds and 140.8 feet. This total "Stop" distance traveled is equivalent to adding Area 1, 2 and 3 in figure 7.

- **"Go" decision**

At 1.0 seconds the driver decides to make no changes and continues at the constant vehicle speed of 30 mph or 44 ft/s. During the yellow light's total phase time of 3.2 seconds the vehicle will travel a distance defined in the first motion example using formula (5.2):

$$\Delta d = \Delta t V_o = 3.2 \text{ s} \times 44 \text{ ft/s} = 140.8 \text{ ft}$$

The total "Go" distance of 140.8 feet is equivalent to adding Area 1, 2 and 4 found in figure 7. We can see that this constant velocity traveled distance during the yellow light phase time is the same as for the driver and vehicle that decided to stop and its total stopping distance.

Using the understanding that the areas under the plotted lines in figure 7 are equal to the distance traveled, we have:

$$\text{Area } 1+2+3 = \text{Area } 1+2+4, \text{ since Area } 3 = \text{Area } 4$$

9.10 The ITE formula example's conclusions

By studying the example we can see that the "Go" vehicle will travel the same distance during the ITE formula's yellow phase time as the "Stop" vehicle will travel to a complete stop. However, the "Stop" vehicle will take 5.4 seconds to complete its traveled distance versus 3.2 seconds for the "Go" vehicle.

We can now draw the conclusion that the ITE formula for the yellow light's total stopping time is actually NOT based on time – the formula is based on *equal distance traveled* for a "Stop" or a "Go" vehicle up to a specific point – the intersection's entry point. This understanding explains the added

“2” in the denominator of the ITE formula’s deceleration term (1.3) since the formula itself violates the basic laws of physics.

Yet, the ITE formula is calculating the traffic light’s yellow phase stopping TIME for a permissive State in the example and we find that the “*one safe stopping distance*” or the “*critical stopping distance*” is therefore the most important formula to understand for the example is as follows:

- If a driver traveling at 30 mph faces a yellow light when he is closer than “*one safe stopping distance*” to the entry of the intersection he must “Go” and continue at the same constant speed without slowing down reaching the intersection’s entry. If the driver is slowing down he might not be able to reach the entry during the time allocated by the ITE formula’s calculated yellow phase time and will thus violate the red light.
- If a driver traveling at 30 mph faces a yellow light when he is farther away than “*one safe stopping distance*” to the entry of the intersection he “shall stop” and the driver is able to stop comfortably and safely based on the input variables for the ITE formula.

Finally, based on the understanding that the ITE formula is not calculating actual deceleration time per the basic laws of physics, we see that the decelerating “Stop” vehicle is still moving at 15 mph which is half the approach speed when the yellow light’s phase time ends and it is taking another 2.2 seconds to come to a complete stop.

The traffic light’s phase change to red and the extra time is not a problem for the stopping vehicle since it still has 24.2 feet (Area 3) to reach the full “*one safe stopping distance*” or the intersection’s entry point. Thus the stopping vehicle will not enter the intersection on a red light. However, what happens, when for instance, a vehicle is within the “*critical stopping distance*” and is slowing down to make a right hand turn? Let us investigate.

10. Fourth motion example: A vehicle making a right hand turn

To be continued...

11. References

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12. Appendix A - Definition of the Yellow Traffic Signal for Vehicles by State

Source ⁵: NCHRP Report 731 "Appendix C"

State	Definition	Steady Yellow Signal Vehicle Code
Alabama	Permissive	Vehicular traffic facing a steady circular yellow or yellow arrow signal is thereby <i>warned</i> that the related green movement is being terminated or that a red indication will be exhibited immediately thereafter.
Alaska	Permissive	<i>No specific information available, assume Uniform Vehicle Code as default.</i>
Arizona	Permissive	Vehicular traffic facing a steady yellow signal is <i>warned</i> by the signal that the related green movement is being terminated or that a red indication will be exhibited immediately thereafter when vehicular traffic shall not enter the intersection.
Arizona	Permissive	Vehicular traffic facing the signal is <i>warned</i> that the red or "STOP" signal will be exhibited immediately thereafter, and vehicular traffic shall not enter the intersection when the red or "STOP" signal is exhibited.
California	Permissive	A driver facing a steady circular yellow or yellow arrow signal is, by that signal, <i>warned</i> that the related green movement is ending or that a red indication will be shown immediately thereafter.
Colorado	Permissive	Vehicular traffic facing a steady circular yellow or yellow arrow signal is thereby <i>warned</i> that the related green movement is being terminated or that a red indication will be exhibited immediately thereafter.
Connecticut	<i>Permissive (Corrected by author)</i>	Vehicular traffic facing a steady yellow signal is thereby <i>warned</i> that the related green movement is being terminated or that a <i>red indication will be exhibited immediately thereafter, when vehicular traffic shall stop</i> before entering the intersection unless so close to the intersection that a stop cannot be made in safety.
Delaware	Permissive	Vehicular traffic facing the circular yellow signal is thereby <i>warned</i> that a red signal for the previously permitted movement will be exhibited immediately thereafter.
Florida	Permissive	Vehicular traffic facing a steady yellow signal is thereby <i>warned</i> that the related green movement is being terminated or that a red indication will be exhibited immediately thereafter when vehicular traffic shall not enter the intersection.
Georgia	Permissive	Traffic, except pedestrians, facing a steady CIRCULAR YELLOW or YELLOW ARROW signal is thereby <i>warned</i> that the related green movement is being terminated or that a red indication will be exhibited immediately thereafter when vehicular traffic shall not enter the intersection.
Hawaii	Permissive	Vehicular traffic facing a steady yellow signal is thereby <i>warned</i> that the related green movement is being terminated or that a red indication will be exhibited immediately thereafter when vehicular traffic shall not enter the intersection.
Idaho	Permissive	A driver facing a steady circular yellow or yellow arrow signal is being <i>warned</i> that the related green movement is ending, or that a red indication will be shown immediately after it.
Illinois	Permissive	Vehicular traffic facing a steady circular yellow or yellow arrow signal is thereby <i>warned</i> that the related green movement is being terminated or that a red indication will be exhibited immediately thereafter.

Indiana	Permissive	Vehicular traffic facing a steady circular yellow or yellow arrow signal is warned that the related green movement is being terminated and that a red indication will be exhibited immediately thereafter.
Iowa	Restrictive	A "steady circular yellow" or "steady yellow arrow" light means vehicular traffic is warned that the related green movement is being terminated and vehicular traffic shall no longer proceed into the intersection and shall stop . If the stop cannot be made in safety, a vehicle may be driven cautiously through the intersection.
Kansas	Permissive	Vehicular traffic facing a steady circular yellow or yellow arrow signal is thereby warned that the related green movement is being terminated or that a red indication will be exhibited immediately thereafter when vehicular traffic shall not enter the intersection.
Kentucky	Permissive	Vehicular traffic facing a steady yellow signal is thereby warned that the related green movement is being terminated or that a red indication will be exhibited immediately thereafter when vehicular traffic shall not enter the intersection.
Louisiana	Permissive (Corrected by author)	Vehicular traffic facing a steady yellow signal alone is thereby warned that the related green signal is being terminated or that a red signal will be exhibited immediately thereafter and such vehicular traffic shall not enter or be crossing the intersection when the red signal is exhibited.
Maine	Permissive	If steady and circular or an arrow, means the operator must take warning that a green light is being terminated or a red light will be exhibited immediately
Maryland	Permissive	Vehicular traffic facing a steady yellow signal is warned that the related green movement is ending or that a red signal, which will prohibit vehicular traffic from entering the intersection, will be shown immediately after the yellow signal
Massachusetts	Permissive	<i>No specific information available, assume Uniform Vehicle Code as default.</i>
Michigan	Restrictive	If the signal exhibits a steady yellow indication, vehicular traffic facing the signal shall stop before entering the nearest crosswalk at the intersection or at a limit line when marked, but if the stop cannot be made in safety, a vehicle may be driven cautiously through the intersection.
Minnesota	Permissive	Vehicular traffic facing a circular yellow signal is thereby warned that the related green movement is being terminated or that a red indication will be exhibited immediately thereafter when vehicular traffic shall not enter the intersection, except for the continued movement allowed by any green arrow indication simultaneously exhibited.
Mississippi	Restrictive	Vehicular traffic facing the signal shall stop before entering the nearest crosswalk at the intersection, but if such stop cannot be made in safety a vehicle may be driven cautiously through the intersection.

Missouri	Permissive	Vehicular traffic facing a steady yellow signal is thereby <i>warned</i> that the related green movement is being terminated or that a red indication will be exhibited immediately thereafter when vehicular traffic shall not enter the intersection.
Montana	Permissive	Vehicular traffic facing a steady circular yellow or yellow arrow signal is <i>warned</i> that the traffic movement permitted by the related green signal is being terminated or that a red signal will be exhibited immediately thereafter. Vehicular traffic may not enter the intersection when the red signal is exhibited after the yellow signal.
Nebraska	<i>Restrictive</i>	Vehicular traffic facing a steady yellow indication is thereby warned that the related green movement is being terminated or that a red indication will be exhibited immediately thereafter when vehicular traffic shall not enter the intersection, and upon display of a steady yellow indication, vehicular traffic <i>shall stop</i> before entering the nearest crosswalk at the intersection, but if such stop cannot be made in safety, a vehicle may be driven cautiously through the intersection.
Nevada	Permissive	Vehicular traffic facing the signal is thereby <i>warned</i> that the related green movement is being terminated or that a steady red indication will be exhibited immediately thereafter, and such vehicular traffic must not enter the intersection when the red signal is exhibited.
New Hampshire	Permissive	Vehicular traffic facing a steady circular yellow or yellow arrow signal is thereby <i>warned</i> that the related green movement is being terminated or that a red indication will be exhibited immediately thereafter when vehicular traffic shall not enter the intersection.
New Jersey	<i>Restrictive</i>	Amber, or yellow, when shown alone following green means <i>traffic to stop</i> before entering the intersection or nearest crosswalk, unless when the amber appears the vehicle or street car is so close to the intersection that with suitable brakes it cannot be stopped in safety. A distance of 50 feet from the intersection is considered a safe stopping distance for a speed of 20 miles per hour, and vehicles and street cars if within that distance when the amber appears alone, and which cannot be stopped with safety, may proceed across the intersection or make a right or left turn unless the turning movement is specifically limited.
New Mexico	Permissive	Vehicular traffic facing the signal is <i>warned</i> that the red signal will be exhibited immediately thereafter and the vehicular traffic shall not enter the intersection when the red signal is exhibited except to turn as hereinafter provided.
New York	Permissive	Traffic, except pedestrians, facing a steady circular yellow signal may enter the intersection; however, said traffic is thereby <i>warned</i> that the related green movement is being terminated or that a red indication will be exhibited immediately thereafter.

North Carolina	Permissive	When a traffic signal is emitting a steady yellow circular light on a traffic signal controlling traffic approaching an intersection or a steady yellow arrow light on a traffic signal controlling traffic turning at an intersection, vehicles facing the yellow light are <i>warned</i> that the related green light is being terminated or a red light will be immediately forthcoming.
North Dakota	Permissive	Vehicular traffic facing a steady circular yellow or yellow arrow indication is thereby <i>warned</i> that the related green movement is being terminated or that a red indication will be exhibited immediately thereafter when vehicular traffic may not enter the intersection.
Ohio	Permissive	Vehicular traffic, streetcars, and trackless trolleys facing a steady circular yellow or yellow arrow signal are thereby <i>warned</i> that the related green movement is being terminated or that a red indication will be exhibited immediately thereafter when vehicular traffic, streetcars, and trackless trolleys shall not enter the intersection.
Oklahoma	Permissive	Vehicular traffic facing a steady circular yellow or yellow arrow signal is thereby <i>warned</i> that the related green movement is being terminated or that a red indication will be exhibited immediately thereafter.
Oregon	<i>Restrictive</i>	A driver facing a steady circular yellow signal light is thereby warned that the related right-of-way is being terminated and that a red or flashing red light will be shown immediately. A driver facing the light <i>shall stop</i> at a clearly marked stop line, but if none, shall stop before entering the marked crosswalk on the near side of the intersection, or if there is no marked crosswalk, then before entering the intersection. If a driver cannot stop in safety, the driver may drive cautiously through the intersection.
Pennsylvania	Permissive	Vehicular traffic facing a steady yellow signal is thereby <i>warned</i> that the related green indication is being terminated or that a red indication will be exhibited immediately thereafter.
Rhode Island	Permissive	Vehicular traffic facing the signal is <i>warned</i> by it that the red or "stop" signal will be exhibited immediately afterwards, and the vehicular traffic shall not enter or be crossing the intersection when the red or "stop" signal is exhibited.
South Carolina	Permissive	Vehicular traffic facing a steady circular yellow or yellow arrow signal is thereby <i>warned</i> that the related green movement is being terminated or that a red indication will be exhibited immediately thereafter.
South Dakota	Permissive	Vehicular traffic facing the signal is thereby <i>warned</i> that the red or "stop" signal will be exhibited immediately thereafter and such vehicular traffic shall not enter the intersection when the red or "stop" signal is exhibited.

Tennessee	Permissive (Corrected by author)	Vehicular traffic facing the signal is <i>warned</i> that the red or "Stop" signal will be exhibited immediately thereafter and that vehicular traffic shall not enter or cross the intersection when the red or "Stop" signal is exhibited.
Texas	Permissive	An operator of a vehicle facing a steady yellow signal is <i>warned</i> by that signal that: (1) movement authorized by a green signal is being terminated; or (2) a red signal is to be given.
Utah	Permissive	The operator of a vehicle facing a steady circular yellow or yellow arrow signal is <i>warned</i> that the allowable movement related to a green signal is being terminated.
Vermont	Permissive	Vehicular traffic facing a steady yellow signal is thereby <i>warned</i> that the related green signal is being terminated or that a red signal will be exhibited immediately thereafter, when vehicular traffic shall not enter the intersection.
Virginia	<i>Restrictive</i>	Steady amber indicates that a change is about to be made in the direction of the moving of traffic. When the amber signal is shown, traffic which has not already entered the intersection, including the crosswalks, <i>shall stop</i> if it is not reasonably safe to continue, but traffic which has already entered the intersection shall continue to move until the intersection has been cleared. The amber signal is a warning that the steady red signal is imminent.
Washington	Permissive	Vehicle operators facing a steady circular yellow or yellow arrow signal are thereby <i>warned</i> that the related green movement is being terminated or that a red indication will be exhibited immediately thereafter when vehicular traffic shall not enter the intersection. Vehicle operators shall stop for pedestrians who are lawfully within the intersection control area as required by <i>RCW 46.61.235(1)</i> .
West Virginia	Permissive (Corrected by author)	Vehicular traffic facing the signal is thereby <i>warned</i> that the red or "stop" signal will be exhibited immediately thereafter and such vehicular traffic shall not enter or be crossing the intersection when the red or "stop" signal is exhibited.
Wisconsin	<i>Restrictive</i>	When shown with or following the green, traffic facing a yellow signal <i>shall stop</i> before entering the intersection unless so close to it that a stop may not be made in safety.
Wyoming	Permissive	Vehicular traffic facing a steady circular yellow or yellow arrow signal is thereby <i>warned</i> that the related green movement is being terminated or that a red indication will be exhibited immediately thereafter.

13. Appendix B - Emergency Stopping Distances and Time Calculations (Rev. 10)

Note: Distances marked in **RED** violate the 30 mph critical stopping distance of 141 feet.

COMMON INPUT DATA	Value							
Perception/Reaction Time, t_{pr} (s):	1.0	(t=1.0 seconds for Emergency Stopping or ITE, Expected Event).						
Air Brake Delay, t_b (s)	0.5	(t _b =0.5+ seconds air brake delay - tractor-trailers, school and public buses)						
Grade, g (%)	0.0	(g is negative for down grade and positive for up hill).						
30 MPH NOMINAL DESIGN ITE, Car Critical Stopping Distance Nonemergency Deceleration Rate								
Design Speed, V (mph):	20	25	30	35	40	45	50	55
AASHTO Coefficient of Friction, f :	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31
Deceleration Rate, a (ft/s ²):	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00
Critical Stopping Distance, d_c (ft):	72.3	103.8	140.7	182.9	230.5	283.5	341.8	405.5
Total Stopping Time, T (s):	3.9	4.7	5.4	6.1	6.9	7.6	8.3	9.1
Car - DRY Pavement								
Design Speed, V (mph):	20	25	30	35	40	45	50	55
AASHTO Coefficient of Friction, f :	0.60	0.60	0.60	0.60	0.60	0.60	0.60	0.60
Deceleration Rate, a (ft/s ²):	19.32	19.32	19.32	19.32	19.32	19.32	19.32	19.32
Emergency Stopping Distance, d (ft):	51.6	71.5	94.1	119.5	147.7	178.7	212.4	248.9
Total Stopping Time, T (s):	2.5	2.9	3.3	3.7	4.0	4.4	4.8	5.2
Car - WET Pavement								
Design Speed, V (mph):	20	25	30	35	40	45	50	55
AASHTO Coefficient of Friction, f :	0.40	0.38	0.35	0.34	0.32	0.31	0.30	0.30
Deceleration Rate, a (ft/s ²):	12.88	12.24	11.27	10.88	10.30	9.98	9.66	9.66
Emergency Stopping Distance, d (ft):	62.7	91.6	129.8	172.3	225.5	283.9	351.3	417.0
Total Stopping Time, T (s):	3.3	4.0	4.9	5.7	6.7	7.6	8.6	9.4
30 MPH NOMINAL DESIGN ITE, Truck/Bus WITH airbrake delay Nonemergency Deceleration Rate								
Design Speed, V (mph):	20	25	30	35	40	45	50	55
AASHTO Coefficient of Friction, f :	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31
Deceleration Rate, a (ft/s ²):	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00
Critical Stopping Distance, d_c (ft):	87.0	122.2	162.7	208.7	259.9	316.6	378.6	446.0
Total Stopping Time, T (s):	4.4	5.2	5.9	6.6	7.4	8.1	8.8	9.6
Truck/Bus, NO air brake delay - DRY Pavement								
Design Speed, V (mph):	20	25	30	35	40	45	50	55
AASHTO Coefficient of Friction, f :	0.46	0.46	0.46	0.46	0.46	0.46	0.46	0.46
Deceleration Rate, a (ft/s ²):	14.80	14.80	14.80	14.80	14.80	14.80	14.80	14.80
Emergency Stopping Distance, d (ft):	58.4	82.1	109.4	140.3	174.9	213.0	254.9	300.3
Total Stopping Time, T (s):	3.0	3.5	4.0	4.5	5.0	5.5	6.0	6.5
Truck, Truck/Bus, NO air brake delay - WET Pavement								
Design Speed, V (mph):	20	25	30	35	40	45	50	55
AASHTO Coefficient of Friction, f :	0.25	0.23	0.21	0.20	0.19	0.18	0.18	0.17
Deceleration Rate, a (ft/s ²):	8.05	7.37	6.83	6.41	6.12	5.86	5.64	5.51
Emergency Stopping Distance, d (ft):	82.7	127.7	185.6	256.6	339.5	437.0	549.7	670.5
Total Stopping Time, T (s):	4.7	6.0	7.5	9.0	10.6	12.3	14.0	15.7
Truck/Bus WITH air brake delay - DRY Pavement								
Design Speed, V (mph):	20	25	30	35	40	45	50	55
AASHTO Coefficient of Friction, f :	0.46	0.46	0.46	0.46	0.46	0.46	0.46	0.46
Deceleration Rate, a (ft/s ²):	14.80	14.80	14.80	14.80	14.80	14.80	14.80	14.80
Emergency Stopping Distance, d (ft):	73.1	100.5	131.4	166.0	204.3	246.1	291.6	340.7
Total Stopping Time, T (s):	3.5	4.0	4.5	5.0	5.5	6.0	6.5	7.0
Truck, Truck/Bus WITH air brake delay - WET Pavement								
Design Speed, V (mph):	20	25	30	35	40	45	50	55
AASHTO Coefficient of Friction, f :	0.25	0.23	0.21	0.20	0.19	0.18	0.18	0.17
Deceleration Rate, a (ft/s ²):	8.05	7.37	6.83	6.41	6.12	5.86	5.64	5.51
Emergency Stopping Distance, d (ft):	97.4	146.1	207.7	282.4	368.9	470.1	586.4	710.9
Total Stopping Time, T (s):	5.2	6.5	8.0	9.5	11.1	12.8	14.5	16.2
Intersection Comparison Data:	East & westbound SW Allen at SW Lombard 30 mph, 141 ft critical stopping distance.							
References								
Air brake delay and stopping distances: ODOT Commercial Driver Manual: http://www.odot.state.or.us/forms/dmv/36.pdf								
Stopping distances: ODOT/OSU February 1997: http://www.oregon.gov/ODOT/HWY/ACCESSMGT/docs/stopdist.pdf ODOT/OSU April 2012: http://cce.oregonstate.edu/sites/cce.oregonstate.edu/files/12-2-stopping-sight-distance.pdf								
<i>Note: Both OSU reports show a typographical error in Table 3B (97 report) and 5A (2012 report) for wet pavement emergency stopping at 50 mph. (Correct value: 351 vs typo. error: 357 feet)</i>								

14. Appendix C - Deceleration Rates and Stopping Distances Comparison (Rev. 5)

Note: Distances marked in **RED** violate the 30 mph critical stopping distance of 141 feet.

(No Air Brake Delay Added)	Maximum Deceleration, a				Vehicle Speed, V	
Federal and State Standards	ft/s ²	g	mph/s	($t=1.0$ s, grade=0%)	25 mph	30 mph
ITE & ODOT "Comfortable Rate":	10.00	0.31	6.82	→ Stopping Distance, ft:	103.8	140.7
ODOT/OSU Emergency Stopping	ft/s ²	g	mph/s	($t=1.0$ s, grade=0%)	25 mph	30 mph
Car Dry Pavement:	19.32	0.60	13.17	→ Stopping Distance, ft:	71.5	94.1
Car Wet Pavement 25 mph:	12.24	0.35	7.68	→ Stopping Distance, ft:	91.6	-
Car Wet Pavement 30 mph:	11.27	0.35	7.68	→ Stopping Distance, ft:	-	129.8
Car Wet Pavement 35 mph:	10.88	0.34	7.42	→ Stopping Distance, ft:	-	-
Truck/Bus Dry Pavement:	14.80	0.46	10.09	→ Stopping Distance, ft:	82.1	109.4
Truck/Bus Wet Pavement 25 mph:	7.37	0.23	5.02	→ Stopping Distance, ft:	127.8	-
Truck/Bus Wet Pavement 30 mph:	6.83	0.21	4.66	→ Stopping Distance, ft:	-	185.5
Truck/Bus Wet Pavement 35 mph:	6.41	0.20	4.37	→ Stopping Distance, ft:	-	-
0.5 s Air Brake Delay Added	Maximum Deceleration, a				Vehicle Speed, V	
ODOT/OSU Emergency Stopping	ft/s ²	g	mph/s	($t=1.5$ s, grade=0%)	25 mph	30 mph
Truck/Bus Dry Pavement:	14.80	0.46	10.09	→ Stopping Distance, ft:	100.5	131.4
Truck/Bus Wet Pavement 25 mph:	7.37	0.23	5.02	→ Stopping Distance, ft:	146.1	-
Truck/Bus Wet Pavement 30 mph:	6.83	0.21	4.66	→ Stopping Distance, ft:	-	207.6
Truck/Bus Wet Pavement 35 mph:	6.41	0.20	4.37	→ Stopping Distance, ft:	-	-
Bus Passenger Standing	ft/s ²	g	mph/s	($t=1.5$ s, grade=0%)	25 mph	30 mph
Maximum Unsupported:	2.25	0.07	1.54	→ Stopping Distance, ft:	353.3	495.5
Loss of Equilibrium:	5.47	0.17	3.73	→ Stopping Distance, ft:	177.8	242.7
Using Handhold:	6.43	0.20	4.39	→ Stopping Distance, ft:	159.5	216.4
Using Vertical Stanchion:	8.69	0.27	5.92	→ Stopping Distance, ft:	132.3	177.3
Bus Passenger Seated	ft/s ²	g	mph/s	($t=1.5$ s, grade=0%)	25 mph	30 mph
Very Uncomfortable:	7.08	0.22	4.83	→ Stopping Distance, ft:	149.9	202.6
Dislodged Untilted Seat:	15.12	0.47	10.31	→ Stopping Distance, ft:	99.5	130.0
Dislodged Tilted Seat:	16.73	0.52	11.41	→ Stopping Distance, ft:	95.2	123.9
NOTE: SW Allen Blvd at SW Lombard Ave, Beaverton Oregon						
Design values: $V=30$mph, $a=10$ ft/s², $t=1.0$ s and $g=0\%$ → 141 feet critical stopping distance (ITE).						
(Distances marked in RED violate the 30 mph critical stopping distance of 141 feet at SW Allen Blvd)						
References						
ODOT and OSU, Emergency stopping distances for wet and dry road conditions:						
February 1997: http://www.oregon.gov/ODOT/HWY/ACCESSMGT/docs/stopdist.pdf						
April 2012: http://cce.oregonstate.edu/sites/cce.oregonstate.edu/files/12-2-stopping-sight-distance.pdf						
Air brake delay and stopping distances:						
ODOT Commercial Driver Manual: http://www.odot.state.or.us/forms/dmv/36.pdf						
Standing and seated bus passenger maximum deceleration rates:						
http://ntl.bts.gov/lib/33000/33300/33313/33313.pdf						
http://ntl.bts.gov/lib/33000/33300/33340/33340.pdf						